## VECTORS

## Vector Addition

Vectors are added tip to tail. Neither the direction or the length of either vector is changed during the process.

Take two vectors, $A$ and $B$ :



OR


Add them together,


OR


The third vector represents the sum of the vectors. This is the resultant vector.
The resultant is always drawn from the tail of the first vector to the tip of the last vector.

If the vectors are in different directions, add them tip to tail.

Motion can be in two directions.


Addition of several vectors.


If vectors are perpendicular to one another ( $\perp$ ), they can be treated independently of one another.


## Analytical Method of Vector Addition

The sum of any two vectors can be determined using trigonometry.
Trigonometry deals with the relationships among angles and sides of triangles.

$$
\begin{aligned}
& \sin \theta=\frac{o p p}{h y p} \\
& \cos \theta=\frac{a d j}{h y p} \\
& \tan \theta=\frac{o p p}{a d j}
\end{aligned}
$$

If two vectors are perpendicular, they form a right angle triangle. The resultant vector will be the hypotenuse. Using Pythagorean theorem, you can calculate the angle and the magnitude of the vector.

## Components of Vectors

It is possible to have one vector and think of it as the resultant vector of two forces. We choose two new vectors in directions that are perpendicular to each other. These are called the component vectors.

Ex. 1 A small boy pulls his wagon, of mass 24 kg giving it a horizontal acceleration of $1.5 \mathrm{~m} / \mathrm{s}^{2}$. If the wagon's handle makes an angle of $40 .{ }^{\circ}$ with the ground while the boy is pulling it, and there is a frictional force of 6.0 N opposing the wagon's motion, with what force is he pulling on the handle of the wagon?

Ex. 2 A boy pushing a 20. kg lawn mower exerts a force of 100. N along the handle. If the handle is elevated $37^{\circ}$ to the horizontal, determine:
A) the component of the applied force that pushes the lawn mower forward
B) the acceleration of the lawn mower, if the frictional force is $60 . \mathrm{N}$
C) the component of the applied force that pushes the lawn mower vertically towards the ground
D) the gravitational force exerted on the lawn mower
E) the total downward force of the mower on the round when pushed
F) the normal force exerted ont eh mower by the ground
$G$ ) the effective coefficient of kinetic friction

## Adding Vectors at Any Angle.

Vector resolution can be used to add any vectors that are not perpendicular to each other. First, break the vector into its components. Sum the vertical components ( $y$ ) and then the horizontal components ( $x$ ). Both the vertical and horizontal components can be added together to obtain the resultant vector.

Ex. 1 Two girls are trying to pull a 20. kg toboggan out of deep snow that provides an opposing force of 8.0 N . Using the following diagram, find the acceleration of the toboggan.


## Equilibrium

When the net force is zero, the object is in equilibrium. According to Newton's laws, the object will not be accelerated because there is no net force on it.

When the vector sum of forces acting at one point is not zero, a force can be applied that will produce equilibrium.
This is the equilibrant force. To find the equilibrant force of two or more forces, first find the resultant force. The equilibrant force is equal in magnitude to the resultant but in the opposite direction.

Ex. 1 Two people are pulling on a tree. What is the equilibrium force exerted by the tree so as to NOT fall over?


## Inclined Planes

When performing inclined plane problems, choose one axis along the incline and the second axis perpendicular to it. Therefore, there is one force parallel to the incline and one force that is perpendicular.

Show all forces acting on an object.


Ex. 1 A box, $m=12 \mathrm{~kg}$, slides down an incline, $\theta=33^{\circ}$. Find the net force acting on the box. What causes this force? Find the acceleration of the box.

Ex. 2 A skier of mass 52 kg skies down a $27^{\circ}$ incline. If $\mu=0.090$, what is the net force on the skier? What is the acceleration of the skier?

## Newton's Third Law

Ex. 1 Two blocks are pushed with a force of 55 N as seen below. If the coefficient of kinetic friction is 0.209, with what force does box $A$ exert on box $B$ ? What force does box $B$ exert on box $A$ (prove mathematically).


